GCE AS/A level

0981/01

S16-0981-01

MATHEMATICS M2
Mechanics

A.M. TUESDAY, 21 June 2016

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle of mass 4 kg moves along the x-axis, starting, when $t = 0$, from the point where $x = 3$. At time t s, its velocity v ms^{-1} is given by

$$v = 12t^2 - 7kt + 1,$$

where k is constant.
When $t = 2$, the displacement of the particle from the origin is 16 m.

(a) Determine the value of k. [5]

(b) Calculate the magnitude of the force acting on the particle when $t = 5$. [4]

2. A particle is projected from horizontal ground with speed 24·5 ms^{-1} in a direction inclined at an angle of 30° above the horizontal.

(a) Calculate the horizontal range of the particle. [6]

(b) Determine the maximum height reached by the particle. [3]

(c) Write down the speed and the direction of motion of the particle as it hits the ground. [1]

3. At time $t = 0$ s, the position vector of an object *A* is $\mathbf{i}$ m and the position vector of another object *B* is $3\mathbf{i}$ m. The constant velocity vector of *A* is $2\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$ ms^{-1} and the constant velocity vector of *B* is $\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$ ms^{-1}. Determine the value of t when *A* and *B* are closest together and find the least distance between *A* and *B*. [9]

4. By burning a charge, a cannon fires a cannon ball of mass 12 kg horizontally. As the cannon ball leaves the cannon, its speed is 600 ms^{-1}. The recoiling part of the cannon has a mass of 1600 kg.

(a) Determine the speed of the recoiling part immediately after the cannon ball leaves the cannon. [3]

(b) Find the energy created by the burning of the charge. State any assumption you have made in your solution. [4]

(c) Calculate the constant force needed to bring the recoiling part to rest in 1·2 m. [2]

5. A particle is attached to one end of a light elastic string of natural length l m and modulus of elasticity λ N. The other end of the string is attached to the ceiling. The particle hangs in equilibrium. The length of the string is 0·95 m when the weight of the particle is 30 N, and 1·15 m when the weight of the particle is 70 N. Find the value of l and the value of λ. [6]

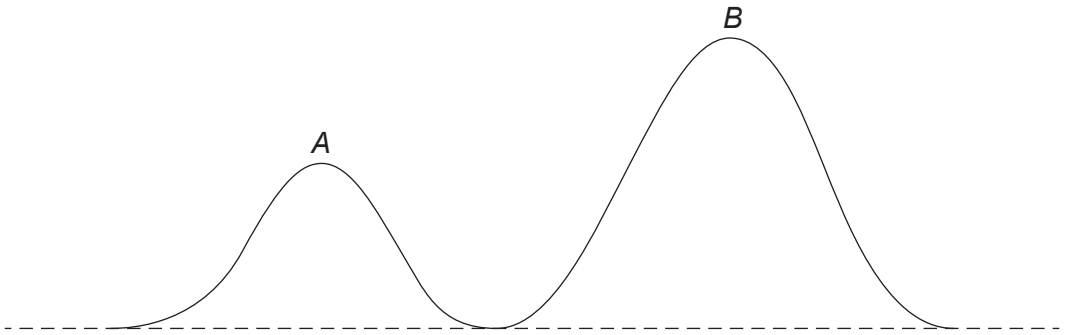
6. A particle moves on a horizontal plane such that its velocity vector $\mathbf{v}$ ms^{-1} at time t s is given by

$$\mathbf{v} = 7\sin 2t\,\mathbf{i} + 6\cos 3t\,\mathbf{j}.$$

(a) Find the acceleration vector of the particle at time t s. [2]

(b) Given that when $t = 0$, the particle has position vector $(0{\cdot}5\mathbf{i} + 3\mathbf{j})$ m, find the position vector of the particle when $t = \frac{\pi}{2}$. [5]

7. The diagram below shows two points *A* and *B* on a mountain bike track.



The heights of *A* and *B* above ground level are 20 m and 22 m respectively. The length of the track between *A* and *B* is 16 m. The resistance to motion of a biker on the track may be modelled by a constant force of magnitude 50 N. The total mass of the biker and his bike is 70 kg. The speed of the biker at *A* is v ms^{-1}. Find the minimum value of v if the biker is to reach *B* without pedalling. [7]

8. A rough circular plate rotates horizontally about a smooth fixed vertical axis through its centre *O*. A point *A* on the plate moves with constant speed v ms^{-1}, where *OA* is 1·6 m. A particle of mass m kg lies on the point *A* on the plate. The coefficient of friction between the particle and the plate is 0·72. Given that the particle remains at the point *A*, find the greatest possible value of v. Hence write down the greatest possible value of the angular velocity of the particle. State clearly your units for the angular velocity. [7]

9. A smooth sphere, with centre *O* and radius 4 m, is fixed. A particle *P*, of mass m, resting on the sphere at its highest point, is given a horizontal speed of magnitude $\sqrt{g}$ ms^{-1}, where g is the magnitude of the acceleration due to gravity. At the instant the line *OP* makes an angle θ with the upwards vertical, the speed of *P* is v ms^{-1}.

(a) Determine an expression for v^2 in terms of g and θ while *P* remains in contact with the sphere. [4]

(b) Find, in terms of m, g and θ, the magnitude of the force exerted by the sphere on *P*. Hence calculate the value of $\cos\theta$ and the value of v^2 when *P* leaves the surface of the sphere. [7]

END OF PAPER

